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PROPAGATION OF A PASSIVE ADMIXTURE FROM AN INSTANTANEOUS LOCALIZED SOURCE IN THE TURBULENT MIXING ZONE IN A PYCNOCLINE

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The dynamics of a passive admixture from an instantaneous localized source in the turbulent mixing zone in a pycnocline is modeled numerically. The source is simulated by specifying the initial distribution of the averaged concentration of the admixture in the form of a finite function that takes a constant value inside a circle of small radius. The results of the calculations show the possibility of situations in which the propagation of the passive admixture is determined to a considerable extent by convective flow generated by the turbulent mixing zone.

Introduction. The study of the evolution of regions of a turbulized fluid — turbulent mixing zones — in homogeneous and stratified media is of interest in connection with the solution of a number of problems of geophysical hydrodynamics [1, 2]. The development of a region of turbulized fluid in a stratified medium is characterized by initial expansion of the mixing zone due to turbulent diffusion, a subsequent cutoff of its vertical growth under the influence of gravity, and the active generation of internal waves. The hydrodynamic aspects of this process were examined in considerable detail in [3–6]. The results of numerical modeling of the dynamics of a passive admixture from an arbitrarily placed, instantaneous localized source in a turbulent mixing zone in a homogeneous and a linearly stratified medium were given in [7]. The source was simulated by specifying the initial distribution of the averaged concentration of the admixture in the form of a finite function that takes a constant value inside a circle of small radius. The significant dependence of the concentration of the admixture on the initial data for this quantity was demonstrated. When the centers of the turbulized region and the localized source do not coincide, the propagation of the admixture is characterized by a shift in the position of the maximum of the averaged concentration toward the center of the turbulized region, but this shift occurs extremely slowly in comparison with the decay of the turbulence.

In the present paper, we consider the problem of the dynamics of a passive admixture from an instantaneous localized source in the turbulent mixing zone in a pycnocline. The possibility of situations in which the propagation of the passive admixture is determined to a considerable extent by convective flow generated by the turbulent mixing zone is demonstrated.

1. Formulation of the Problem. Basic Equations. To describe the propagation of a passive admixture in the turbulent mixing zone in a stratified medium, we use the following system of averaged equations of motion, continuity, incompressibility, and transport of the concentration Θ of the passive admixture, balance of turbulence energy e, transfer of the dissipation rate ε, and Reynolds shear stress $\langle u'v'\rangle$:

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho_0}\frac{\partial\langle p_1\rangle}{\partial x} - \frac{\partial}{\partial x}\langle u'^2\rangle - \frac{\partial}{\partial y}\langle u'v'\rangle,$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho_0}\frac{\partial\langle p_1\rangle}{\partial y} - \frac{\partial}{\partial x}\langle u'v'\rangle - \frac{\partial}{\partial y}\langle v'^2\rangle - \frac{g\langle\rho_1\rangle}{\rho_0},$$

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$$\frac{\partial\langle\rho_1\rangle}{\partial t}+U\frac{\partial\langle\rho_1\rangle}{\partial x}+V\frac{\partial\langle\rho_1\rangle}{\partial y}+V\frac{d\rho_s}{dy}=-\frac{\partial}{\partial x}\langle u'\rho'\rangle-\frac{\partial}{\partial y}\langle v'\rho'\rangle,$$

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0,\quad \frac{\partial\Theta}{\partial t}+U\frac{\partial\Theta}{\partial x}+V\frac{\partial\Theta}{\partial y}=\frac{\partial}{\partial x}K_{\theta x}\frac{\partial\Theta}{\partial x}+\frac{\partial}{\partial y}K_{\theta y}\frac{\partial\Theta}{\partial y},$$

$$\frac{\partial e}{\partial t}+U\frac{\partial e}{\partial x}+V\frac{\partial e}{\partial y}=\frac{\partial}{\partial x}K_{ex}\frac{\partial e}{\partial x}+\frac{\partial}{\partial y}K_{ey}\frac{\partial e}{\partial y}+P+G-\varepsilon, \tag{1.1}$$

$$\frac{\partial\varepsilon}{\partial t}+U\frac{\partial\varepsilon}{\partial x}+V\frac{\partial\varepsilon}{\partial y}=\frac{\partial}{\partial x}K_{\varepsilon x}\frac{\partial\varepsilon}{\partial x}+\frac{\partial}{\partial y}K_{\varepsilon y}\frac{\partial\varepsilon}{\partial y}+C_{\varepsilon 1}\frac{\varepsilon}{e}(P+G)-C_{\varepsilon 2}\frac{\varepsilon^2}{e},$$

$$\frac{\partial\langle u'v'\rangle}{\partial t}+U\frac{\partial\langle u'v'\rangle}{\partial x}+V\frac{\partial\langle u'v'\rangle}{\partial y}=\frac{\partial}{\partial x}K_{ex}\frac{\partial\langle u'v'\rangle}{\partial x}+\frac{\partial}{\partial y}K_{ey}\frac{\partial\langle u'v'\rangle}{\partial y}+(1-C_2)P_{12}+(1-C_3)G_{12}-C_1\frac{\varepsilon}{e}\langle u'v'\rangle.$$

In Eqs. (1.1), U and V are the components of the velocity of the averaged motion in the directions of the x and y axes, respectively (the x axis is horizontal and the y axis is vertically upward, against the force of gravity), p_1 is the departure of the pressure from the hydrostatic pressure $\rho_s(y)$, due to stratification, g is the acceleration of gravity; $\langle\rho_1\rangle$ is the averaged density defect: $\rho_1=\rho-\rho_s$, $\rho_s=\rho_s(y)$ is the density of the undisturbed fluid: $d\rho_s/dy\leqslant 0$ (stable stratification), $\rho_0=\rho_s(Y)$ is the characteristic density of the undisturbed fluid corresponding to $y=Y$, the pulsation components are primed, angle brackets denote averaging, the fluid density is assumed to be a linear function of temperature; stratification is assumed to be weak, and the Oberbeck–Boussinesq approximation is used; terms containing a cofactor in the form of the coefficient of laminar viscosity or diffusion are omitted under the assumption of smallness.

Model of Turbulent Motion. The normal Reynolds stresses $\langle u_i'^2\rangle$ ($i=1,2$) are determined from the isotropic approximations [8]

$$\frac{\langle u_i'u_j'\rangle}{e}=\frac{2}{3}\delta_{ij}+\frac{1-C_2}{C_1}\Big(\frac{P_{ij}}{\varepsilon}-\frac{2}{3}\delta_{ij}\frac{P}{\varepsilon}\Big)+\frac{1-C_3}{C_1}\Big(\frac{G_{ij}}{\varepsilon}-\frac{2}{3}\delta_{ij}\frac{G}{\varepsilon}\Big); \tag{1.2}$$

$$P_{ij}=-\Big\{\langle u_i'u_k'\rangle\frac{\partial U_j}{\partial x_k}+\langle u_j'u_k'\rangle\frac{\partial U_i}{\partial x_k}\Big\},\quad G_{ij}=\frac{1}{\rho_0}\big(\langle u_i'\rho'\rangle g_j+\langle u_j'\rho'\rangle g_i\big),$$

$$\mathbf{g}=(0,-g,0),\quad U_1=U,\quad U_2=V,\quad 2P=P_{ii},\quad 2G=G_{ii}.$$

To find the components of the flux vector $\langle u_i'\rho'\rangle$ ($i=1,2$), as in [7], we use the following consequences of the approximation of local equilibrium:

$$-\langle u'\rho'\rangle=K_{\rho x}\frac{\partial\langle\rho\rangle}{\partial x},\qquad -\langle v'\rho'\rangle=K_{\rho y}\frac{\partial\langle\rho\rangle}{\partial y}.$$

The coefficients of turbulent viscosity K_{ex}, K_{ey}, $K_{\varepsilon x}$, and $K_{\varepsilon y}$ and of diffusion $K_{\rho x}$, $K_{\rho y}$, $K_{\theta x}$, and $K_{\theta y}$ are determined from the equations

$$K_{ex}=C_s\frac{e\langle u'^2\rangle}{\varepsilon},\quad K_{\varepsilon x}=\frac{K_{ex}}{\sigma},\quad K_{ey}=C_s\frac{e\langle v'^2\rangle}{\varepsilon},\quad K_{\varepsilon y}=\frac{K_{ey}}{\sigma},$$

$$K_{\rho x}=K_{\theta x}=\frac{\langle u'^2\rangle e}{C_{1t}\varepsilon},\quad K_{\rho y}=K_{\theta y}=(1/C_{1t}\varepsilon)\Big(e\langle v'^2\rangle\Big/\Big(1-2\frac{g}{\rho_0}\frac{1-C_{2t}}{C_tC_{1t}}\frac{e^2}{\varepsilon^2}\frac{\partial\langle\rho\rangle}{\partial y}\Big)\Big).$$

We took the following empirical constants of the model [9, 10]: $C_{\varepsilon 1}=1.45$, $C_{\varepsilon 2}=1.90$, $\sigma=1.3$, $C_1=2.2$, $C_2=C_3=0.55$, $C_s=0.25$, $C_t=1.25$, $C_{1t}=3.2$, and $C_{2t}=0.5$.

Initial and Boundary Conditions. We took the following boundary and initial conditions for system (1.1):

$$U=V=\langle\rho_1\rangle=e=\varepsilon=\Theta=\langle u'v'\rangle=0,\quad r^2=x^2+y^2\to\infty,\quad t\geqslant 0,$$

$$e(0,x,y)=e_0(r),\quad \varepsilon(0,x,y)=\varepsilon_0(r),\quad \Theta(0,x,y)=\Theta_0(x,y),\quad r^2\leqslant R^2,$$

$$e(0,x,y)=\varepsilon(0,x,y)=\Theta(0,x,y)=0,\qquad r^2\geqslant R^2,$$

$$\langle\rho_1\rangle=U=V=\langle u'v'\rangle=0,\quad -\infty<x<\infty,\quad -\infty<y<\infty,\quad t=0.$$

Here $e_0(r)$ and $\varepsilon_0(r)$ are the self-similar distributions corresponding to a homogeneous fluid (finite, bell-shaped functions) and R is the radius of the region of turbulized fluid at the initial time. The function $\Theta_0(x,y)$ was set equal to $\Theta^0 = \text{const}$ inside a circle Ω^0 of radius $R_0 < R$ and zero outside this circle. This simulated an instantaneous localized source of the admixture. In the numerical solution of the problem, the zero boundary conditions corresponding to $r \to \infty$ were brought in to the boundaries of a sufficiently large rectangle.

The density distribution in the pycnocline was given by the equation

$$\rho_s(y) = \rho_0(1 - a\beta \tanh((y - Y)/\beta)),$$

where a, β, and Y are positive parameters.

Normalization. The variables of the problem are made dimensionless using the scales of length R, velocity $U_0 = \sqrt{e(0,0,0)}$, and averaged concentration Θ^0. We also use the representation $\langle\rho_1\rangle^* = \langle\rho_1\rangle/aR\rho_0$, where $a = -(1/\rho_0)d\rho_s/dy$ at $y = Y$. As a result, the quantity $4\pi^2/\mathrm{Fr}^2$ appears in the dimensionless equations instead of g, where $\mathrm{Fr} = U_0T/R$ is the density Froude number ($T = 2\pi/\sqrt{ag}$ is the Väisälä–Brunt period). Henceforth the dimensionless variables are marked by an asterisk.

Algorithm for Solving the Problem and Its Testing. The finite-difference algorithm is based on the application of methods of separation with respect to spatial variables, has first-order approximation in time and second-order approximation in the spatial variables, and was presented in [6].

The mathematical model for this study differs from the model used in [7] to describe flow in the case of a linearly stratified medium by the way the coefficients of turbulent viscosity are represented [in [7] they were obtained as a consequence of the isotropic approximations (1.2)]. This is because the model of [7] yields unsatisfactory results in describing the wave pattern of flow in a pycnocline. The numerical model developed was tested on the problem of the evolution of a zero-momentum wake in a linearly stratified medium. The results of a comparison of calculated data on the behavior of the turbulence characteristics in the wake with the experimental data of Lin and Pao were given in [11]. There we showed that the calculated patterns of internal waves generated by the wake in the pycnocline are consistent with the known experimental data [12].

2. Calculation Results. To analyze the turbulent diffusion of a passive impurity from a localized source in the turbulent mixing zone in a pycnocline, we performed a series of numerical experiments, in which we varied the position of the source of admixture within the turbulized region (the parameters x_0 and y_0), the width of the transitional layer of the pycnocline (the parameter β), and the mutual arrangement of the turbulized region and the fluid layer with the maximum vertical density gradient (the parameter Y).

The origin of coordinates coincides with the center of the turbulized region. By analogy with the linear stratification [7], we consider the following variants of the coordinates of the center of the circle Ω^0: $x_0 = y_0 = 0$ (variant No. 1), $x_0 = 0$ and $y_0 = 0.57R$ (variant No. 2), $x_0 = y_0 = 0.57R$ (variant No. 3), and $x_0 = y_0 = 0.28R$ (variant No. 4). The main results are given for $\mathrm{Fr} = 4.7$.

The calculations were made on nonuniform orthogonal grids, which bunch together in the vicinity of the turbulent mixing zone and Ω^0, having 120×100 nodes. Here the grid analog of Ω^0 was an approximate simulation of a circle with a diameter of six cells, $R_0 = 0.17R$. To estimate the accuracy, we made calculations on a grid with 240×200 nodes and horizontal and vertical cell sizes half as large in the vicinity of the turbulent mixing zone. The resulting deviations did not exceed 5% in the uniform grid norm.

The results of the calculations in a pycnocline with $Y = 0$ and $\beta = 0.19R$ are shown in Fig. 1 ($t/T = 3$). Streamlines of $\psi/(U_0R) = \text{const}$ are shown in Fig. 1a; curves 1–9 correspond to levels of $1.9 \cdot 10^{-3}$, $1.5 \cdot 10^{-3}$, $9.4 \cdot 10^{-4}$, $3.8 \cdot 10^{-4}$, 0, $-3.8 \cdot 10^{-4}$, $-9.4 \cdot 10^{-4}$, $-1.5 \cdot 10^{-3}$, and $-1.9 \cdot 10^{-3}$. Isolines of the turbulence energy $e/e_m(t) = \text{const}$, $e_m(t) = \max\limits_{x,y} e(t,x,y)$ (Fig. 1b) are given at levels of 0.01, 0.1, and then up to 0.9 with an interval of 0.1. Isolines of the averaged concentration of the passive admixture, $\Theta/\Theta_m(t) = \text{const}$, $\Theta_m(t) = \max\limits_{x,y} \Theta(t,x,y)$ (Figs. 1c and 2), are given for variant No. 3 ($x_0 = y_0 = 0.57R$) of the initial position of the source; here Fig. 2, in contrast to Fig. 1c, corresponds to a pycnocline with $Y = 0.57R$ and $\beta = 0.19R$. In Figs. 1c and 2, the point marks the node of the grid region at which the averaged concentration reaches a maximum; the dashed curve marks the boundary of the turbulized region, determined by the equation $e(t,x,y) = 0.01e_m(t)$; the levels are the same as in Fig. 1b. It is seen that the spot of the passive admixture,

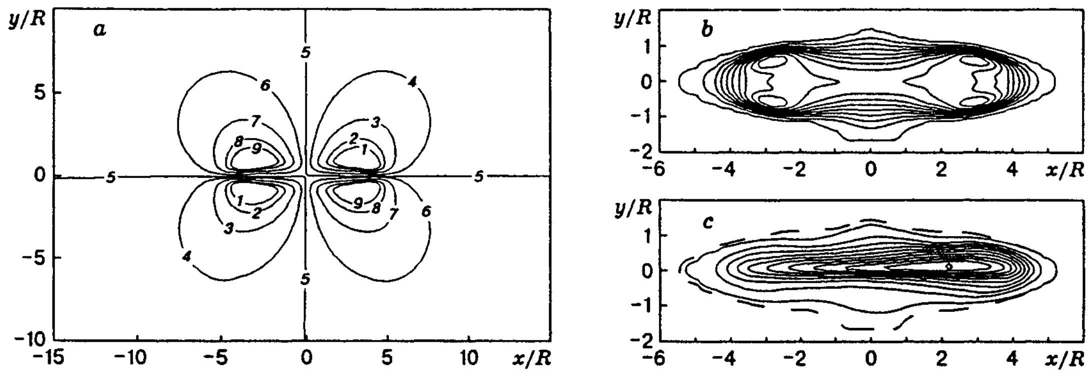

Fig. 1

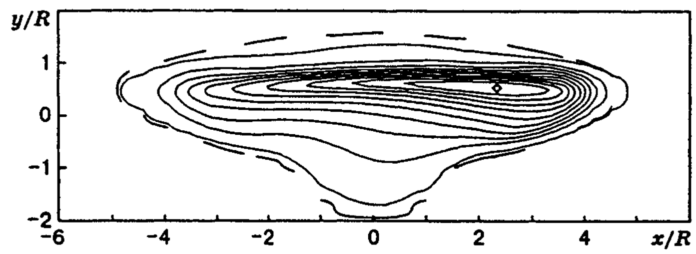

Fig. 2

initially concentrated in a small round region within the turbulized zone, eventually spreads out in the form of tongues along the horizontal interlayer of fluid with the maximum vertical density gradient, duplicating the shape of the turbulized region. The patterns of outflow of the turbulized region obtained in these calculations are similar to those observed in laboratory experiments [13], in which the dynamics of spots of partially mixed fluid in a thin-layered stratified medium was investigated.

Curves 1–5 in Fig. 3 illustrate the time variation of the maximum averaged concentration $\Theta_m^*(t)$. Curves 1–3 correspond to calculations in a pycnocline with $Y = 0$ and $\beta = 0.19R$ (variant Nos. 1–3 of the position of Ω^0); curves 4 and 5 correspond to linear stratification (variant No. 3: $x_0 = y_0 = 0.57R$). The difference in the behavior of curves 1–3 can be explained by the nonuniformity of the distribution of the turbulent diffusion coefficients $K_{\theta x}$ and $K_{\theta y}$. Curves 6–8 describe the behavior of the characteristic turbulence energy $e_c(t)/U_0^2 = e(t,0,0)/U_0^2 = e_c^*(t)$ at the center of the region of turbulent mixing for nonlinear (curve 6) and linear (curves 7 and 8) density distributions of an unperturbed fluid with depth. Here curves 5 and 8 were obtained in calculations from the model of [7]. It is seen that the two models yield similar results in the case of linear stratification. Over the time interval $t/T \in [0, 10]$ the turbulence energy decreases by four orders of magnitude.

In Fig. 4 we show the time variation of the abscissa $x_m(t)$ and ordinate $y_m(t)$ of the grid node at which the concentration maximum $\Theta_m^*(t) = \Theta^*(t, x_m, y_m)$ is reached. Here curves 1–3 were obtained for variant No. 3 of the initial position of the admixture source ($x_0 = y_0 = 0.57R$): curve 1 refers to linear stratification, curve 2 to a pycnocline with $Y = 0$ and $\beta = 0.57R$, and curve 3 to a "narrow" pycnocline ($Y = 0$ and $\beta = 0.19R$). In contrast to curve 3, curves 4 and 5 correspond to variant Nos. 2 and 4, respectively, of the coordinates of the center of Ω^0 (curves 4 refer to $x_0 = 0$ and $y_0 = 0.57R$ and curve 5 refers to $x_0 = y_0 = 0.28R$; $Y = 0$ and $\beta = 0.19R$) and curves 6 and 7 correspond to the new versions of the position of the fluid layer with the largest vertical density gradients $\rho_s(y)$ relative to the center of the turbulized region (curve 6 refers to $Y = 0.19R$ and curve 7 refers to $Y = 0.57R$; $\beta = 0.19R$ and $x_0 = y_0 = 0.57R$).

In the case of linear stratification, the value of $x_m(t)$ is nearly constant over the entire time interval examined (curve 1 in Fig. 4a). In the case of a pycnocline, this quantity grows intensely for all t/T except for a brief initial stage of development of the turbulized region when, as in a homogeneous fluid, turbulent

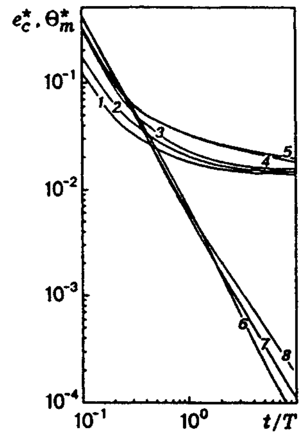

Fig. 3

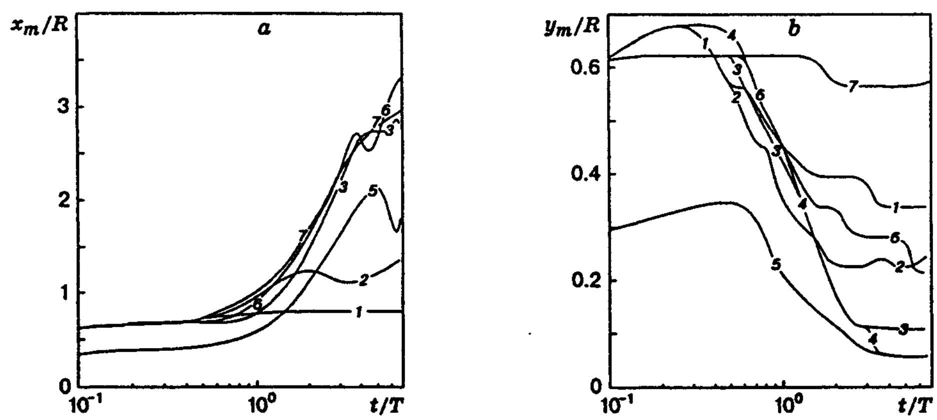

Fig. 4

diffusion is dominant. The growth of $x_m(t)$ is associated with a property of the wave pattern of flow in the pycnocline [6, 12], namely, the formation in each quadrant of the (x, y) plane of a convective vortex of high intensity, which migrates along the horizontal axis in the direction of increasing $|x|$ (Fig. 1a). Such behavior of $x_m(t)$ is typical of all the versions of the position of the admixture source examined (curves 3 and 5) and values of the parameter Y (curves 3, 6, and 7). At the same time, curve 2, which corresponds to a pycnocline with a broader transitional layer, is close to curve 1, which refers to linear stratification.

In Fig. 5, as a supplement to Fig. 4a, using the example of a calculation with the parameters $x_0 = y_0 = 0.57R$, $Y = 0$, and $\beta = 0.19R$, we compare the time dependences of the grid analogs of the abscissa x_s of the maximum point of the stream function (curve 1), the abscissa x_m of the concentration maximum (curves 2 and 3), and the horizontal dimensions L_x and l_x of the turbulized region (curves 4–7). Here curves 4 and 5 illustrate the behavior of the horizontal size of the turbulized zone, calculated from the equation $e(t, L_x, 0) = 0.01e(t, 0, 0)$, and curves 6 and 7 were obtained from the equation $e(t, l_x, 0) = 0.5e(t, 0, 0)$. Curves 3, 5, and 7 correspond to calculations on a 240 × 200 grid. These data show that, as in the case of a homogeneous fluid [7], turbulent diffusion leads to a shift in the position of the concentration maximum to the coordinate origin. The convective flow induced by the collapse of the turbulent mixing zone causes intense horizontal transport of the admixture. Figures 4a and 5 reflect the interaction between turbulent diffusion and convective transport. The behavior of curves 6 and 7 in Fig. 5 is the result of the generation of turbulence energy by the convective flow.

The calculations show that in the case of linear stratification, the coordinate $y_m(t)$ increases slightly in the initial stage of development of the turbulized zone and then declines (curve 1 in Fig. 4b). Even at

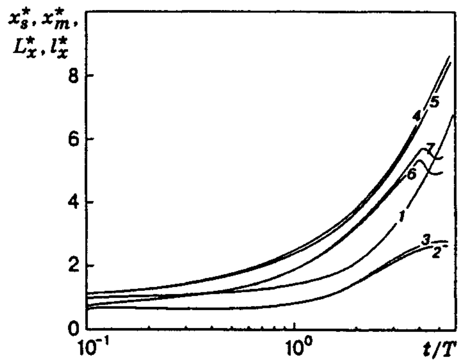

Fig. 5

$t/T \geqslant 4$, however, this quantity differs from zero (like x_m for an asymmetrical placement of the source), which indicates that the averaged concentration of the passive admixture "remembers" the features of its initial distribution. Behavior similar to that of y_m also occurs in a pycnocline with a broad transitional layer (curve 2). At the same time, the numerical experimental data show that in the evolution of the turbulized region in a pycnocline, the averaged concentration of the passive admixture at late times reaches its maximum in interlayers of fluid with the largest vertical density gradients. In the case of a pycnocline with a narrow transitional layer, in particular, $y_m(t)$ at $t/T \geqslant 1$ depends essentially on the value of Y (curves 3, 6, and 7 in Fig. 4b).

Thus, in all cases in which x_0 and y_0 differ from zero, the concentration of the passive admixture reaches a maximum at a considerable distance from the origin of coordinates, not only at late times but also for $t/T \geqslant 4$.

Calculations were also made for a larger Froude number (Fr = 22.1). The data obtained are in qualitative agreement with those given here for Fr = 4.7, but, as in the evolution of a turbulized region in a linearly stratified fluid [7], the action of stratification is manifested later, i.e., for larger $t_* = (t/T)\mathrm{Fr} = tU_0/R$.

Memory effects in the propagation of an admixture in a nonisothermal, free turbulent flow are fairly well known [14, 15]. They are also observed in the problem of the dynamics of a turbulent mixing zone in homogeneous and linearly stratified fluids [7]. The present results demonstrate the important role of convective flow in the propagation of a passive admixture from a localized source in the turbulent mixing zone in a pycnocline.

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REFERENCES

1. J. S. Turner, *Buoyancy Effects in Fluids*, Cambridge Univ. Press (1973).
2. A. S. Monin and R. V. Ozmidov, *Turbulence in the Ocean* [in Russian], Gidrometeoizdat, Leningrad (1981) [D. Reidel, Dordrecht, Holland (1984)].
3. O. F. Vasil'ev, B. G. Kuznetsov, Yu. M. Lytkin, and G. G. Chernykh, "Development of a region of turbulized fluid in a stratified medium," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3, 45–52 (1974).
4. A. M. Trokhan and Yu. D. Chashechkin, "Generation of internal waves in a stratified fluid by a hydrodynamically linear source (two-dimensional problem)," in: *Theory of Wave Diffraction and*

Propagation: Abstracts of VII Symp. on Wave Diffraction and Propagation (Rostov-on-Don, 1977), Vol. 3, Izd. Akad. Nauk SSSR, Moscow (1977), pp. 186–189.

5. Yu. M. Lytkin and G. G. Chernykh, "Flow similarity with respect to the density Froude number and energy balance in the evolution of a turbulent mixing zone in a stratified medium," *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **47** (1980), pp. 70–89.

6. O. F. Voropaeva and G. G. Chernykh, "Evolution of the turbulent mixing zone in a fluid with nonlinear stratification," in: *Simulation in Mechanics* (Collected scientific papers) [in Russian], Computer Center–Inst. of Theor. and Appl. Mech., Sib. Div., Acad. of Sci. of the USSR, **3(20)**, No. 5, (1989), pp. 3–29.

7. O. F. Voropaeva, Yu. D. Chashechkin, and G. G. Chernykh, "Diffusion of a passive admixture from an instantaneous localized source in a turbulent mixing zone," *Dokl. Ross. Akad. Nauk*, **356**, No. 6, 759–762 (1997).

8. W. Rodi, "Examples of calculation methods for flow and mixing in stratified fluids," *J. Geophys. Res.*, **92**, No. C5, 5305–5328 (1987).

9. M. M. Gibson and B. E. Launder, "On the calculation of horizontal, turbulent, free shear flows under gravitational influence," *Trans. ASME*, **C98**, No. 1, 81–87 (1976).

10. W. Rodi, *Turbulence Models and Their Application in Hydraulics*, Univ. of Karlsruhe, Karlsruhe (1981).

11. O. F. Voropaeva and G. G. Chernykh, "Numerical model of the dynamics of a momentum-free turbulent wake in a pycnocline," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 3, 69–86 (1997).

12. H. E. Gilreath and A. Brandt, "Experiments on the generation of internal waves in a stratified fluid," *AIAA J.*, **23**, 693–700 (1985).

13. V. A. Popov, "Development of a region of partially mixed fluid in a thin-layered stratified medium," *Izv. Akad. Nauk SSSR, Fiz. Atmosf. Okeana*, **22**, No. 4, 389–394 (1986).

14. Yu. M. Dmitrenko, V. L. Zhdanov, and B. A. Kolovandin, "Influence of initial conditions on the structure of a nonisothermal, axisymmetric turbulent wake," Preprint, Inst. of Heat and Mass Transfer, Acad. of Sci. of the Belorus. SSR, Minsk (1985).

15. V. I. Bukreev, A. G. Demenkov, V. A. Kostomakha, and G. G. Chernykh, "Heat propagation from a linear source in a plane turbulent medium," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 5, 115–126 (1996).